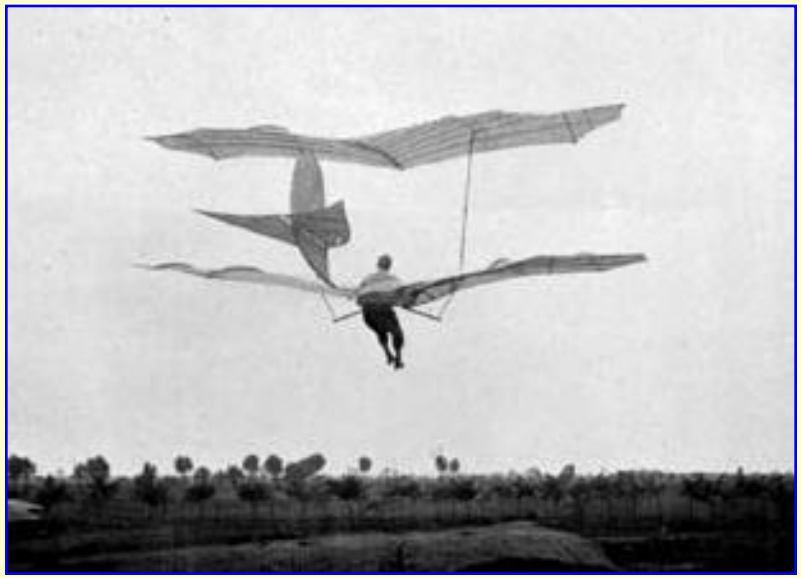
Aerodynamics Naïve #1– derivation of the ping-pong polar diagrams – a simplifying yet intuitive analogy

by George Lungu

- This tutorial contains a effective yet simple and intuitive to understand analogy for deriving the "ping-pong polar diagrams". There is nothing else here except evaluating the forces involved in a ping-pong paddle hitting a series of incoming balls at an angle.
- Though the premises of this derivation are just partially correct, the results will later prove strikingly similar to the real polar diagrams. The insight one can get out of this very simple derivation is extremely important since it shows how the lift and drag come from the same source, namely deflecting air molecules downwards. It is also a contains a crude component of viscosity effects.
- Aircraft professionals, please stay out of this. Many things here might sound blasphemous to you, yet this is my own personal take on the topic and I believe one can do advanced aerodynamic modeling without going through the standard years of ordeal for getting a minimum background.

Some brief history:

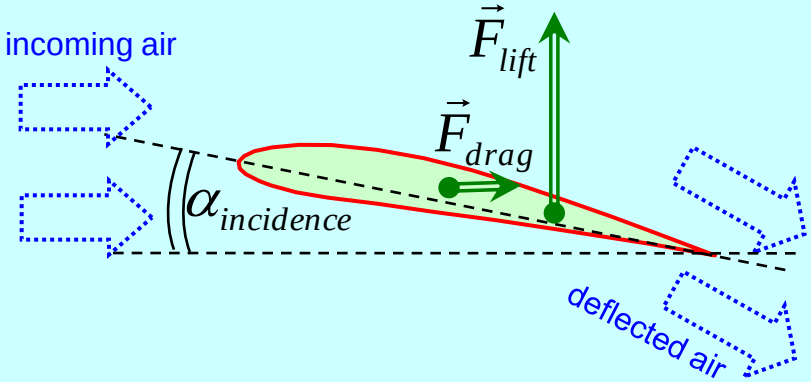
- There is little remembrance today about Otto Lilienthal, the pioneer who had more to do with the early advancement of flight than the Wright brothers.
- You can search more for yourself, but this guy flew a lot of self developed hang gliders more than a century ago in Germany, sacrificing his own life in the process. He also experimentally studied the properties of flying surfaces at various shapes, angles and speeds. He did this rigorously and systematically creating the famous "polar diagrams" which, until today remain one of the most useful tools in aircraft design.

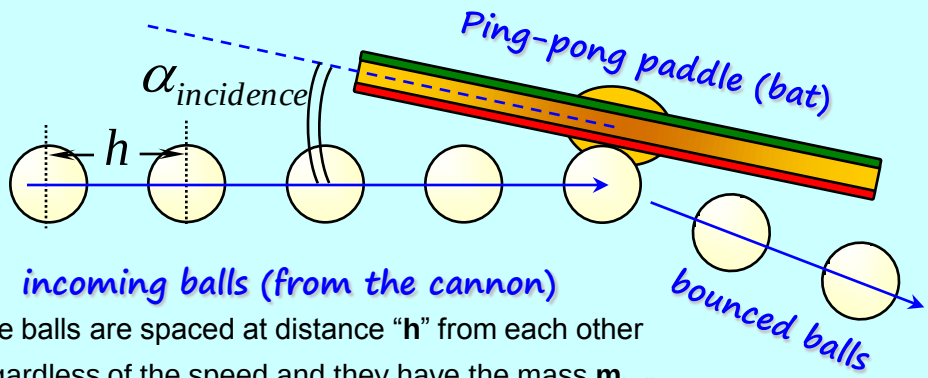


Otto Lilienthal during the landing of one of his gliders in 1895.

The ping-pong analogy:

- Let's imagine we have a fixed Ping-Pong paddle making a slight angle with the horizontal, which models the wing airfoil.
- Let's now assume that we have a very precise cannon shooting balls horizontally. We are able to adjust the speed of the balls but the shooting frequency will be controlled by the cannon so that the distance between consecutive balls stays constant (h) . The mass and the distance between the balls signify the air density.
- Also let's assume that the balls are uniformly coated with a viscous "goo", which is not a regular glue, instead gives the ball the tendency to stick to the paddle just a little so that there is a small tangential force between the paddle and the colliding ball.
- That sticking force, proportional to the speed will model the viscosity of the air.
- This first model will only simulate what happens on the bottom of the paddle, handling the processes on the top cannot be done to any extent by using this overly simplifying analogy.



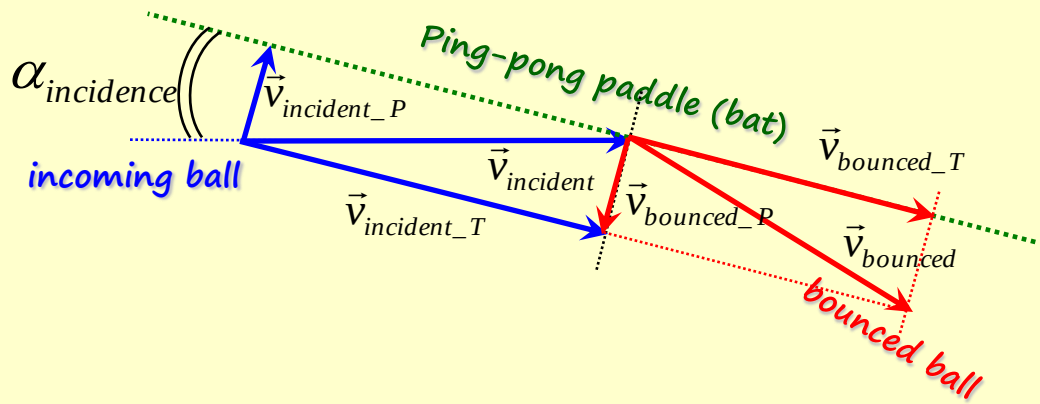


The balls are spaced at distance "**h**" from each other regardless of the speed and they have the mass $\mathbf{m_{ball}}$

Estimating the forces involved in the collision between the paddle and the balls:

- Let's first estimate how the speed of the ball will be affected by the collision with the immobile paddle.

- The incoming speed can be decomposed in two perpendicular components, one tangential to the bat which is not affected by the collision but it will be reduced by the viscosity (stickiness of the ball to the paddle) and one perpendicular to the bat surface which will keep its magnitude and direction after the collision but will reverse its sense.



$$v_{incident\_P} = -v_{bounced\_P} = v_{incident} \cdot \sin(\alpha_{incidence})$$

- The linear momentum of the ball is defined as the product between the mass of the ball and it's speed. It is a vector. The linear momentum change (absolute value) the ball experiences perpendicular to the bat can be written as:

$$\Delta p_P = p_{bounced\_P} - p_{incident\_P} = m_{ball} \cdot v_{bounced\_P} - m_{ball} \cdot v_{incident\_P} = 2 \cdot m_{ball} \cdot v_{incident} \cdot \sin(\alpha_{incidence})$$

- We can calculate the linear momentum change of the bat (if we release the bat for an instant). By using the law of linear momentum conservation we can say that the bat experiences a momentum change equal in magnitude and direction but of opposite sense to that of the ball (we drop the minus sign since this is an absolute value calculation):

$$\Delta p_{P\_bat} = 2 \cdot m_{ball} \cdot v_{incident} \cdot \sin(\alpha_{incidence})$$

- We can calculate the average force exerted on the paddle by the incoming balls using Newton second law and knowing that the spacing between two consecutive balls is constant and equal to "h":

$$v_{incident} = \frac{h}{\Delta t_{collision}} \qquad F_{P\_average\_bat} = \frac{\Delta p_{P\_bat}}{\Delta t_{collision}} = \frac{2 \cdot m_{ball} \cdot v_{incident} \cdot \sin(\alpha_{incidence})}{\Delta t_{collision}}$$

$$F_{P\_average\_bat} = \frac{\Delta P_{P\_bat}}{\Delta t_{collision}} = \frac{2 \cdot m_{ball}}{h} \cdot v_{incident}^2 \cdot \sin(\alpha_{incidence})$$

Lets call $\frac{2 \cdot m_{ball}}{h} = K_{inertial}$ and this is proportional to an analogous air density

We assumed a unit bat area but let's introduce the area in the final formula. The final average force perpendicular to the bat is an inertial force and it has the final formula:

$$F_{P\_bat} = A_{bat} \cdot K_{inertial} \cdot v_{incident}^2 \cdot \sin(\alpha_{incidence})$$

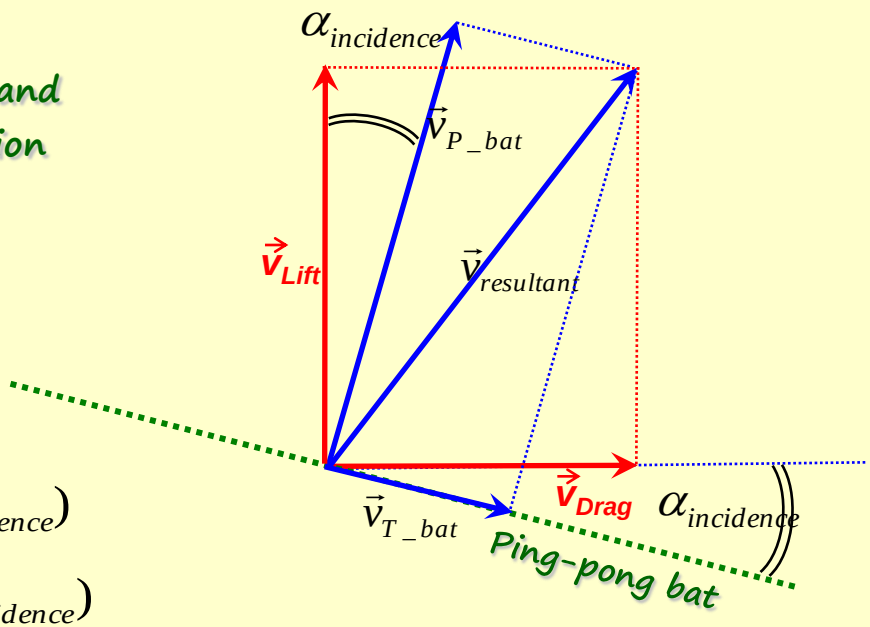
We know already the tangential speed of the incoming balls to the bat. Let's simplify things and use a general assumption valid for viscous friction that the viscous friction force is proportional to product of the wing (bat) area and the relative tangential speed difference between the bat and the fluid (series of balls). Of course we need to introduce a constant of proportionality, $K_{viscous}$:

The final average force tangential to the bat is: $F_{T\_bat} = A_{bat} \cdot K_{viscous} \cdot \underbrace{v_{incident} \cdot \cos(\alpha_{incidence})}_{(v_{incident\_T})}$

Estimating the lift and the drag forces:

- Now that we have the perpendicular (normal) force and the tangential force, a simple trigonometric manipulation will lead us to the final formulas for the lift and drag.

- The lift is a vertical force and the drag is a horizontal force.
- We can write the following formulas:



$$\begin{cases} F_{Lift} = F_{P\_bat} \cdot \cos(\alpha_{incidence}) - F_{T\_bat} \cdot \sin(\alpha_{incidence}) \\ F_{Drag} = F_{P\_bat} \cdot \sin(\alpha_{incidence}) + F_{T\_bat} \cdot \cos(\alpha_{incidence}) \end{cases}$$

And the final lift and drag forces are (we dropped the index "incident" and "incidence"):

$$F_{Lift} = A_{bat} \cdot \left(K_{inertial} \cdot v^2 - K_{viscous} \cdot v\right) \cdot \sin(\alpha) \cdot \cos(\alpha)$$

$$F_{Drag} = A_{bat} \cdot \left(K_{inertial} \cdot v^2 \cdot \sin^2(\alpha) + K_{viscous} \cdot v \cdot \cos^2(\alpha)\right)$$

The lift and drag coefficients and their ratio:

- In order to calculate and chart the polar coordinate we need to calculate the lift and drag coefficients which are essentially the lift and drag forces normalized to the area of the wing, the density of the air and the square of the speed. Their definitions are (we replaced A_{bat} with A_{wing}) :

$$c_{Lift} = 2 \cdot \frac{F_{Lift}}{A_{wing} \cdot \rho \cdot v^2}$$

$$c_{Drag} = 2 \cdot \frac{F_{Drag}}{A_{wing} \cdot \rho \cdot v^2}$$

The ratio of the two coefficients is the gliding ratio and it's a useful formula. The final lift and drag coefficients and their ratio are:

$$c_{Lift} = \frac{2 \cdot \left(K_{inertial} \cdot v - K_{viscous}\right) \cdot \sin(\alpha) \cdot \cos(\alpha)}{\rho \cdot v}$$

$$c_{Drag} = \frac{2 \cdot \left(K_{inertial} \cdot v \cdot \sin^2(\alpha) + K_{viscous} \cdot \cos^2(\alpha)\right)}{\rho \cdot v}$$

$$\frac{c_{Lift}}{c_{Drag}} = \frac{\left(K_{inertial} \cdot v - K_{viscous}\right) \cdot \sin(\alpha) \cdot \cos(\alpha)}{K_{inertial} \cdot v \cdot \sin^2(\alpha) + K_{viscous} \cdot \cos^2(\alpha)}$$

Conclusions

- A ping-pong analogy was used to estimate the lift and drag forces and the corresponding coefficients of a flat, thin airfoil in a "ping-pong paddle" model. These formulas will later be used to chart the polar diagrams of the same foil.
- Even though the assumptions involved are overly simplified in nature and have seemingly not that much to do with real aerodynamics, the basic physics of the paddle ball interaction were modeled correctly and we will later see that the resulting polar diagrams will look pretty similar to the real ones for a thin flat foil.
- The simplified assumptions left no room for any estimate of the angular momentum on a wing or modeling what happen on the top of the wing (including stall).
- We will later use these formulas to model a virtual glider.

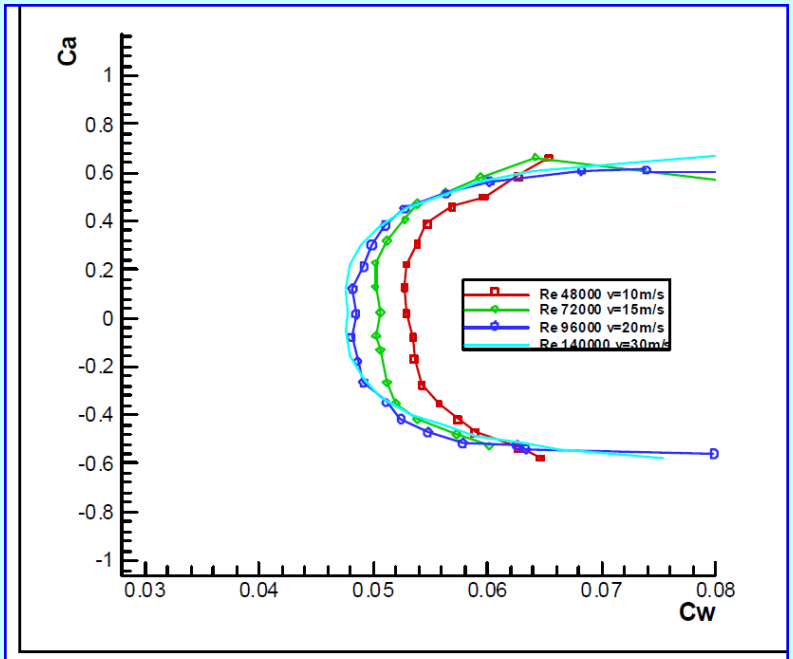


Figure 1.2-1: Cl-Cd diagram WillsWing Carbon 1.2

Example polar coordinates for a performance hang glider