Excel anaglyph stereoscopy #3 – 3D-2D perspective conversion formulas

by George Lungu

- While the previous section introduced the reader to the general principles of anaglyph stereoscopy, this section explains the geometry and calculations behind implementing this method in an Excel model.

- There are some subtle differences between the method as used in photography and the method as applied in computer graphics. To realize that, one can simply think of the fact that a picture has theoretically only one plane perpendicular to the axis of the camera where the image is in perfect focus. In computer graphics all the picture is theoretically in focus. We can easily emulate a real camera but there is no gain in doing that. In computer graphics we want to be able to watch any area of an image and the focus should stay perfect. This is a clear example of humans selectively imitating nature (we try to imitate only the positive aspects and not the drawbacks).

- By the same token if in real life, we try to focus on something really close in front of our nose for instance, our eyes will cross and the image that lays behind (farther) that point will split (due to the very converging vision). In our 3D computer model we will try to model the near field image by having to watch it with a large eye conversion, yet while looking at a point in the far field of the image the eye conversion will be much lower.

- It's just like taking a very large 2D array of pictures of a scene with the proper focus and lens convergence (when the space between the two red and cyan cameras is constant the converging angle drops as we look farther in the distance). After that, tile up the central areas of each picture (where the focus and convergence is perfect) and create a large composite picture which, provided there are no border artifacts between tiles, is perfect. Without all the effort and scissors we can do this in software.

Let's outline the ideas from the previous page:

- The vision system of a human or a stereoscopic camera will image a scene optimizing focus and convergence to a small chosen area of the image then apply the same settings to the whole image.
- In the case of a human the eyes and the brain will constantly adjust focus and convergence while the human scans the image.
- A computer model (including ours) must optimize the settings of focus and "eye convergence" for every point in the scene across all the u-v space (the monitor surface).
- I will add one more detail here, namely our model must be built with an adjustable but constant "eye span" (distance between red-cyan cameras). Increasing or decreasing this distance must increase or decrease the 3D effect to a level that the user feels comfortable watching (a more pronounced 3D effect will force the user to increase the "cross-eye" convergence but it will be more exciting to watch.

A useful device analogy:

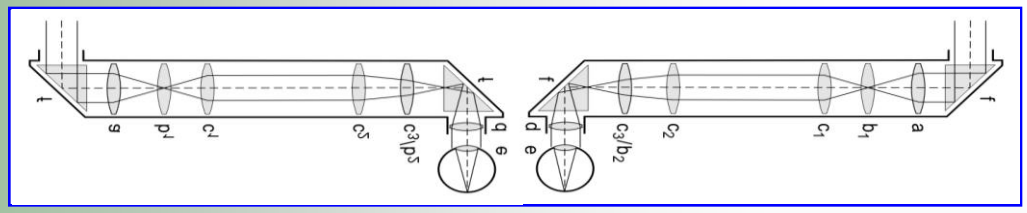
-During the Great War and after, there was a popular artillery optical instrument called the "Scissors Scope" which was essentially a pair of matched periscopes which could symmetrically swivel or pivot around the axes of the eyepieces.

- Using lenses and optical prisms to deflect light, this device allows a soldier not only safely watch the front from the trenches but also to greatly increase his sense of perspective and even perform distance measurements by optical triangulation.

The scissor (telemetry) scope:

- Math and geometry in programming has no meaning unless they can be directly tied to a physical device. It is very important to find real life analogies before delving into formulas, otherwise we might never be able to solve certain problems.

-By adjusting the angle between the two half-scopes, the viewer can effectively extend his vision sideways and increase the effective distance between his eyes. All the image angles at the objective lenses are transmitted unchanged to the eyes (it's just like the width of the head of the observer is be increased – like the head of a hammer shark).



What does this device do?

The device has two functions:

1. It collects two images from two different positions (with adjustable span between images)
2. After the collection it brings the images close together one to each eye individually.

Why do we need to collect images from different points (angles)? – a straight (frontal) look at an object collects information about the face of an object while stereoscopic view collects information about the face and the sides of the object. The amount of information collected from the sides of the object is proportional to the scope opening (i.e. the distance between the objective lenses)

Why we need to bring the images together? – we need this in order to analyze the images at the same time and create ourselves a sense of perspective associated with increased visual information.

What do we borrow from the scissor scope analogy?

- Our model will contain two parameters: eye-span and collection-span. Both of these will be adjustable input parameters and serve to change the perspective setup for various models (a flight simulator for instance will require a large collection span since the distances in the scene are large).
- The larger the collection span, the more stereoscopic effect we will get for a given distance in front of the viewer.
- Due to the limited cross-eye convergence angle of a human observer (around 45 degrees), for any given collection span there is a minimum distance the viewer will be able to perceive the object as a unit. Objects closer than that will visually split.

Due to a large eye span, a hammerhead shark has very good stereoscopic vision with close to 360 degrees of field of view

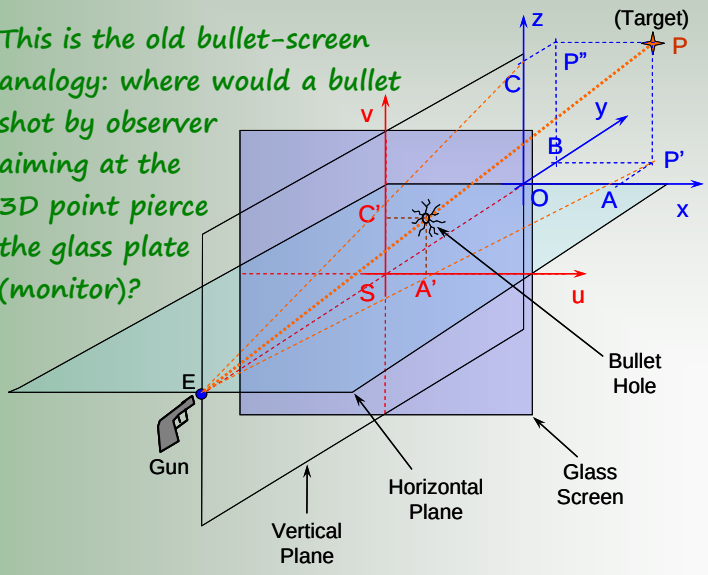
Various conventions:

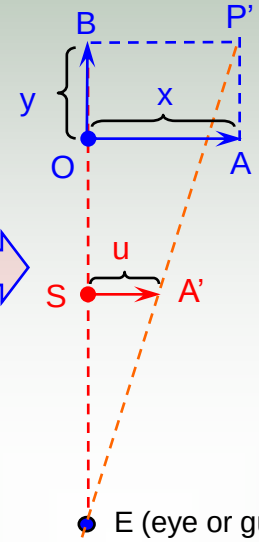
- Just like most the other models on this site the y-axis will face forward, the x-axis will face to the right and the z-axis will face up.
- The u-axis has the same direction and sense with the x axis. The v-axis has the same direction and sense with the z axis.
- Let's assume that the eyes of the observer are placed along a horizontal line parallel to the x-axis. Because of this we can assume with a good approximation that all the stereoscopic angle transformations can be restricted to calculations in the x-y plane (the plane will also contain the u-axis) and we will equate both right and left v-coordinates with the same v-coordinate calculated using the old formula.
- On the other hand, calculating exact left and right v-coordinates is not hard but unequal v-coordinates reaching the eyes create unwanted vertical misalignments unpleasant to the eyes.

The stereoscopic 3D-2D perspective conversion setups:

- We used the old setup below when we derived the plain 3D-2D perspective conversion formulas:

This is the old bullet-screen analogy: where would a bullet shot by observer aiming at the 3D point pierce the glass plate (monitor)?





using triangle similarity we arrived to the final formulas for the u-v coordinate pair:

$$u = \frac{x \cdot ES}{ES + SO + y}$$

$$v = \frac{z \cdot ES}{ES + SO + y}$$

Where:

- ES is the distance between eye and screen
- SO is the distance between the screen and the origin of the 3D object system of coordinates.

- Here is the new stereoscopic setup:

Using cameras placed at a certain distance between each other (let's call it "collection_span"):

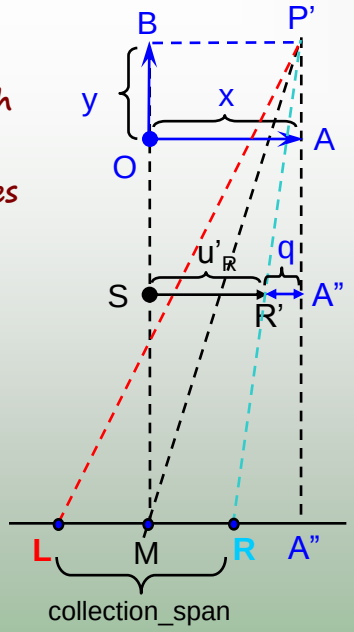
-P' is the projection of the target point P (of coordinates (x,y,z) on the plane x-y.

- L is the position of the left camera (red).

- R is the position of the right camera (cyan).

- M is the midpoint between cameras (center of symmetry – formerly known as E).

- MS is the distance between the middle point between the plane of the cameras and the screen.



- SO is the distance between the screen and the origin of the 3D object system of coordinates.

-Again, the distance between the cameras (LR) is named "collection_span".

- Later we will introduce the "eye-span' which will be a smaller distance than the "collection_span".

- u'_R is the u coordinate of point P in the u-v space. Later we will calculate u_R which is the u coordinate after the rays are brought to the eye level.

Deriving the stereoscopic 3D-2D perspective conversion formulas:

Triangles P'A'R' and P'A''R are similar triangles therefore we can write the following relations of proportionality:

$$\frac{|A'P'|}{|A''P'|}=\frac{|R'A'|}{|RA''|} \Rightarrow |R'A'|=|RA''|\cdot\frac{|A'P'|}{|A''P'|}$$

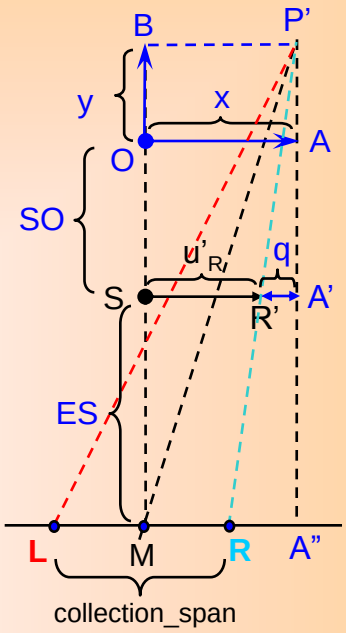
We can also write the following relations:

$$\begin{cases} |RA''| = x-\dfrac{Collection\_span}{2} \\ |A'P'| = |SO|+y \\ |A''P'| = |ES|+|SO|+y \end{cases}$$

$$\Rightarrow |R'A'| = q = \left(x-\frac{Collection\_span}{2}\right)\cdot\frac{SO+y}{ES+SO+y}$$

Knowing the value of "q" we can express the value of u'$_R$ as:

$$\Rightarrow u'_R = x-q = \frac{Collection\_span\cdot(SO+y)}{2\cdot(ES+SO+y)}-\frac{x\cdot ES}{ES+SO+y}$$



Shifting the right (cyan) ray from the collection point to the right eye:

Keeping all the angles constant we perform a linear shift to the left:

$$u_R = u'_R - |E_{Right}R| = u'_R - \frac{(Collection\_span - Eye\_span)}{2}$$

$$\Rightarrow u_R = \frac{Eye\_span}{2}-\frac{(2x+Collection\_span)\cdot ES}{2\cdot(ES+SO+y)}$$

We can find u_L for the left eye (red) following a similar derivation or just by considering Eye_span and Collection_span vectors and changing their sign:

$$\Rightarrow u_L = -\frac{Eye\_span}{2}-\frac{(2x-Collection\_span)\cdot ES}{2\cdot(ES+SO+y)}$$