An Animated Linear Feedback Shift Register (LFSR) as a Pseudo Random Pattern Generator in Excel 2003- Part#1

by George Lungu

- This tutorial is not meant to be a theoretical introduction to random number generation but a mere cook-book recipe for building such a generator as an animated Fibonacci type LFSR in Microsoft Excel 2003.
- A very sketchy theoretical introduction is given after which a 14-bit long Fibonacci type LFSR device is implemented in a worksheet having feedback which generates a maximum length sequence ($2^{14}-1$).
- This type of device is covered in most undergraduate curricula in schools such as Electrical Engineering, Computer Engineering and Computer Science.
- Being a while since I myself studied this device in school and having a very limited time budget I briskly refreshed my memory from Wikipedia:

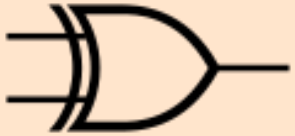
(http://en.wikipedia.org/wiki/Linear_feedback_shift_register)

and used their Fibonacci setup and their formula table for a maximum sequence 14-bit LFSR.

An outline from Wikipedia:

- A linear feedback shift register (LFSR) is a shift register whose input bit at any time is a linear function of its previous state.
- Applications of LFSR's include generating pseudo-random numbers, pseudo-noise sequences, fast digital counters, and whitening sequences. Both hardware and software implementations of LFSR's are common.

- The mostly used linear function of single bits is XOR, thus normally it is a shift register whose input bit is driven by the exclusive-or (XOR) of some bits of the overall shift register value.



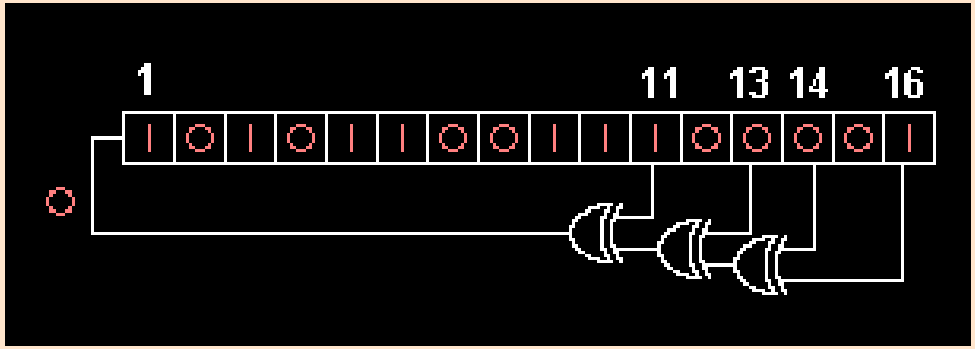
The symbol of a XOR gate and its truth table

INPUT A	INPUT B	OUTPUT A XOR B
0	0	0
0	1	1
1	0	1
1	1	0

- The initial value of the LFSR is called the seed and because the operation of the register is deterministic, the stream of values produced by the register is completely determined by its current (or previous) state.

- Likewise, because the register has a finite number of possible states, it must eventually enter a repeating cycle. However, an LFSR with a well-chosen feedback function can produce a sequence of bits which appears random and which has a very long cycle.

Fibonacci LFSR's:

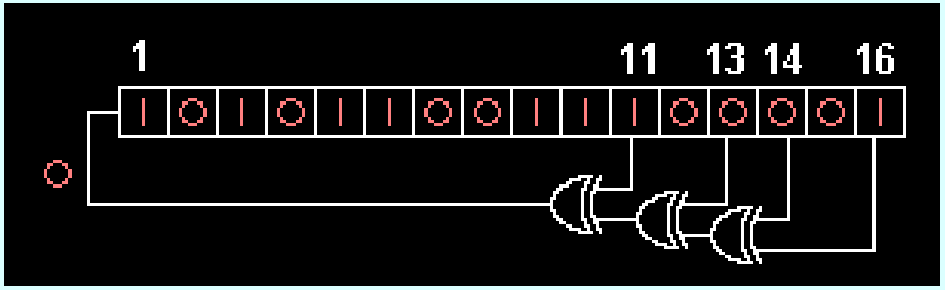


-The bit positions that affect the next state are called the taps. In the diagram the taps are [16,14,13,11]. The rightmost bit of the LFSR is called the output bit. The taps are XOR'd sequentially with the output bit and then fed back into the leftmost bit. The sequence of bits in the rightmost position is called the output stream.

- The bits in the LFSR state which influence the input are called taps (white in the diagram).

-A maximum-length LFSR produces an m-sequence (i.e. it cycles through all possible 2n – 1 states within the shift register except the state where all bits are zero), unless it contains all zeros, in which case it will never change. The sequence of numbers generated by an LFSR can be considered a binary numeral system just as valid as Gray code or the natural binary code

-The arrangement of taps for feedback in an LFSR can be expressed in finite field arithmetic as a polynomial mod 2. This means that the coefficients of the polynomial must be 1's or 0's. - This is called the feedback polynomial or characteristic polynomial. For example, if the taps are at the 16th, 14th, 13th and 11th bits (as shown above), the feedback polynomial is:



$$x^{16} + x^{14} + x^{13} + x^{11} + 1$$

Some polynomials for maximal LFSRs:

Bits	Feedback polynomial	Period
n		$2^n - 1$
2	$x^2 + x + 1$	3
3	$x^3 + x^2 + 1$	7
4	$x^4 + x^3 + 1$	15
5	$x^5 + x^3 + 1$	31
6	$x^6 + x^5 + 1$	63
7	$x^7 + x^6 + 1$	127
8	$x^8 + x^6 + x^5 + x^4 + 1$	255
9	$x^9 + x^5 + 1$	511
10	$x^{10} + x^7 + 1$	1023
11	$x^{11} + x^9 + 1$	2047
12	$x^{12} + x^{11} + x^{10} + x^4 + 1$	4095
13	$x^{13} + x^{12} + x^{11} + x^8 + 1$	8191
14	$x^{14} + x^{13} + x^{12} + x^2 + 1$	16383
15	$x^{15} + x^{14} + 1$	32767

-The table to the left lists maximal-length polynomials for shift-register lengths up to 15. Note that more than one maximal-length polynomial may exist for any given shift-register length.

Some polynomials for maximal LFSRs:

-Ones and zeroes occur in 'runs'. The output stream 0110100, for example consists of five runs of lengths 1,2,1,1,2, in order. In one period of a maximal LFSR, 2n -1 runs occur (for example, a six bit LFSR will have 32 runs). Exactly 1/2 of these runs will be one bit long, 1/4 will be two bits long, up to a single run of zeroes n - 1 bits long, and a single run of ones n bits long. This distribution almost equals the statistical expectation value for a truly random sequence. However, the probability of finding exactly this distribution in a sample of a truly random sequence is rather low.

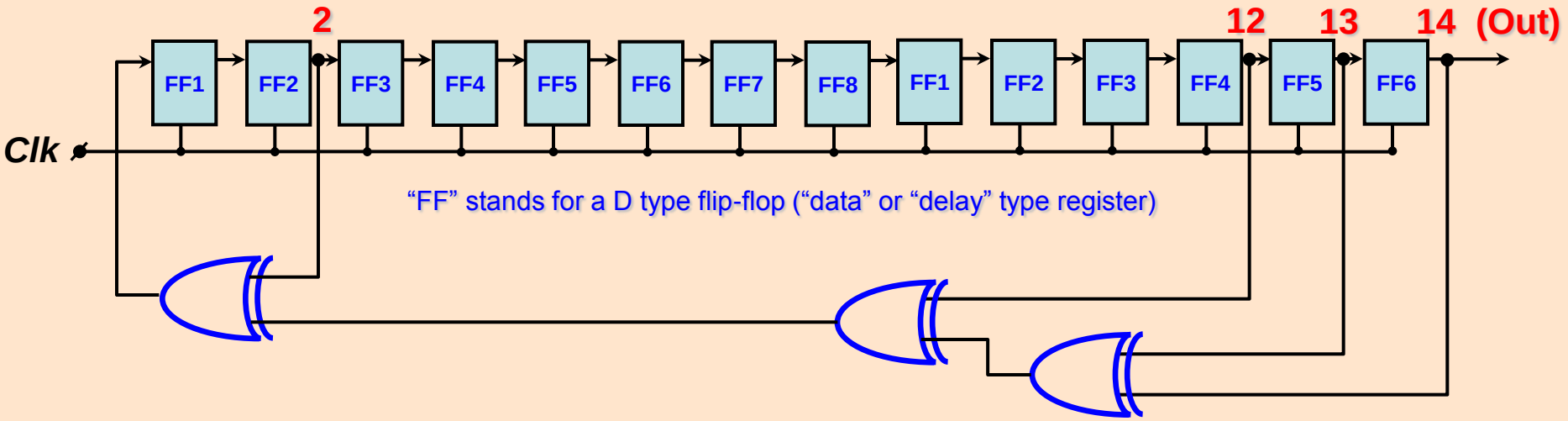
-LFSR output streams are deterministic. If you know the present state, you can predict the next state. This is not possible with truly random events.

-The output stream is reversible; an LFSR with mirrored taps will cycle through the output sequence in reverse order.

Let's consider a maximal length 14 bit LFSR :

- Why we chose 14 bit? The answer is that we need a lively animated model. Handling and charting $2^{14}-1 = 16383$ points of data must be fairly fast in Excel. The number of points is also large enough to give the user a feel of data randomness
- Let's take the polynomial formula from the table in the previous page and implement the following schematic.

$$x^{14} + x^{13} + x^{12} + x^2 + 1$$



- The D type flip-flops in the above diagram will delay the incoming data by one clock period. The output will take the value of the input during the rising edge of the clock and it will hold it until the new rising edge occurs. In short this flip-flop will delay the input data by one clock period and synchronize it with the clock, which means the output will transition only during the clock rising edges.
- In other words the flip-flop chain in the diagram above is just a shift register

Dynamic Excel implementation outline:

-This will be a dynamic (sequential) animated model run by a "copy-paste" type of macro used in many of the previous models on this blog.

- Two different macros will be used, a "Reset" macro and a "Run-Pause" macro.

- The reset macro will clear all the simulation history and set the flip-flops using the seed value. I will arbitrarily choose "1000000000000" as seed but you can use any other number except "0000000000000" (make sure there is at least a "1" in the group).

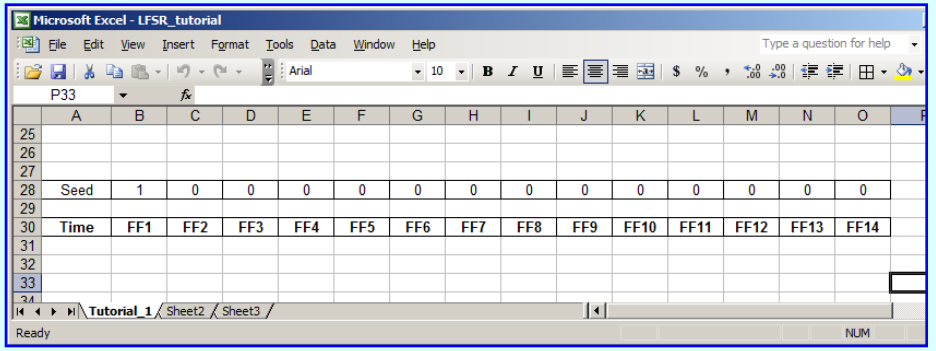
- We will create a instantaneous spreadsheet calculation for the device for 30 time steps (on 30 rows) and during each cycle, the "Run_Pause" macro, will copy 17000 time steps of history information of the model and paste it 30 cells down (in the past). The macro will do this operation repeatedly in a loop creating continuously time rolling simulation environment in which 16353 time steps of information are charted in real time. The chart will be dynamically updated and you will be able to see the effect of any change in shape of the waveforms.

- The worksheet will therefore have an instantaneous calculation area (30 rows) and a historical area (2^14-31 = 14353 rows) underneath.

Excel implementation:

- Open a new file and name it "LFSR_tutorial"

- Name the first worksheet "Tutorial_1" and in it, fill out the labels as seen in the snapshot to the right.



- In the range B28:O28 type in the seed bits as constants. The seed can be any combination of "zeros" and "ones" but make sure there are not all zeros.

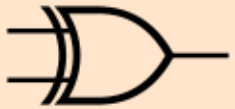
Creating an exclusive OR (XOR) Excel custom function:

- Open the VBA editor and create (insert) a module
- In the newly created module type in the code to the right

```
Function EXOR(A, B)
    If (A = 0 And B = 1) Or (A = 1 And B = 0) Then
        EXOR = 1
        Else
        EXOR = 0
        End If
End Function
```

- It seems that "XOR" is a reserved word in VBA since the program would not allow its use without flagging an error.

The symbol of a XOR gate and its truth table



INPUT A	INPUT B	OUTPUT A XOR B
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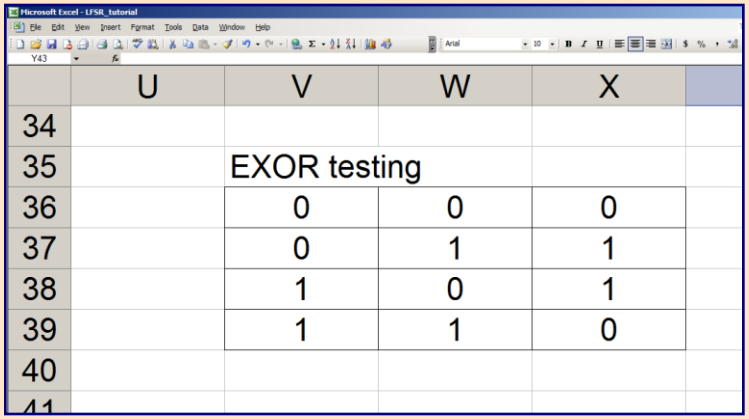
1	1	0

Testing the exclusive OR (XOR) Excel custom function:

- In cell X36 type "=EXOR(V36, W36) and drag-copy X36 to cell X39
- Fill out range V36:W39 with the following combination of 0's and 1's (all possible combinations)
- It seems that the function works as expected from a exclusive OR, namely returning a "0" (low) whenever the inputs are equal and returning a "1" (high) whenever the inputs are different



	U	V	W	X
34				
35		EXOR testing		
36		0	0	0
37		0	1	1
38		1	0	1
39		1	1	0
40				