An Animated Linear Feedback Shift Register (LFSR) as a Pseudo Random Pattern Generator in Excel 2003 – Part#3

by George Lungu

- In the previous section, a table type implementation of a LFSR was created
- This section converts the 14-bit binary pseudo-random number series into a fractional series of numbers between 0 and 1 then it display this series of numbers against an offset version or itself on a 2D scatter chart using a variable offset.

Converting the binary 14-bit numbers into fractional numbers in the range [0,1]:

- We will use the following formula:

$$n = \frac{a_{13}}{2} + \frac{a_{12}}{2^2} + \frac{a_{11}}{2^3} + \frac{a_{10}}{2^4} + \frac{a_9}{2^5} + \frac{a_8}{2^6} + \frac{a_7}{2^7} + \frac{a_6}{2^8} + \frac{a_5}{2^9} + \frac{a_4}{2^{10}} + \frac{a_3}{2^{11}} + \frac{a_2}{2^{12}} + \frac{a_1}{2^{13}} + \frac{a_0}{2^{14}}$$

- In the above formula "n" represents the number in decimal-fractional form in the range [0,1) and a_0, a_1, a_2, ..., a_{13} represent the bits of the original binary number with a_0 being the least significant bit (LSB) and a_{13} being the most significant bit (MSB).

Worksheet implementation:

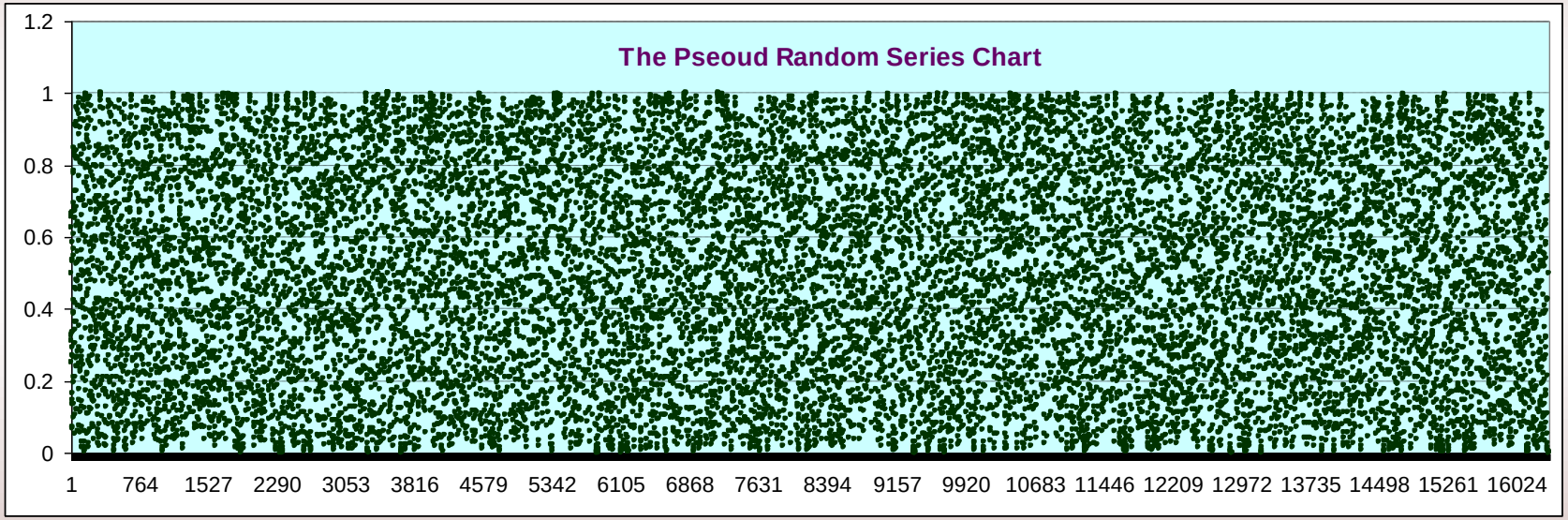
- First let's rename the second worksheet (Tutorial_2) as "Tutorial_2&3
- In cell Q31 type the following formula:

Q31: "=B31/2+C31/2^2+D31/2^3+E31/2^4+F31/2^5+G31/2^6+H31/2^7+I31/2^8+J31/2^9+K31/2^10+L31/2^11+M31/2^12+N31/2^13+O31/2^14"

- Drag-copy cell Q31 down to cell Q16414

Charting the pseudo random series:

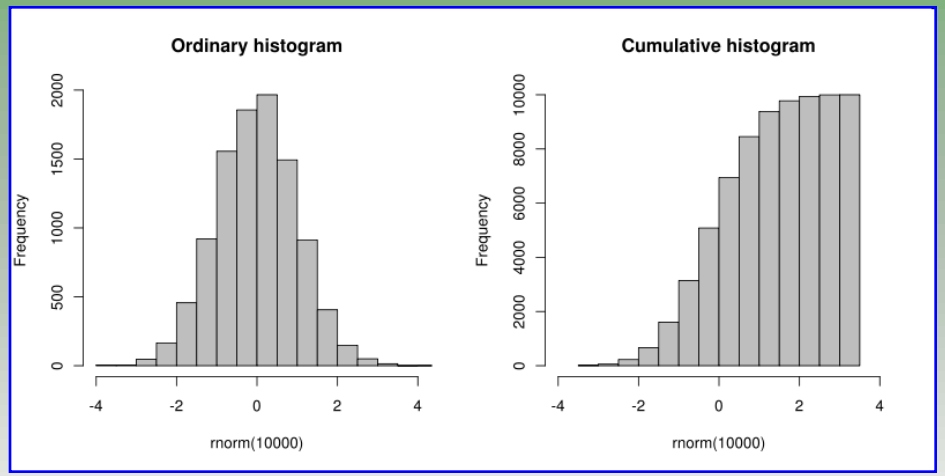
- Select range Q31:Q16141 then: Insert => Chart => Line => Finish
- Select the series on the chart by double clicking it, then reduce the marker size to "3", change the color of the marker to your preference, change the shape to solid round and choose "Line => None". You could change the color of the plot area if you wish by double clicking it.
- The numbers composing the series appear to be uniformly distributed between 0 and 1



Creating a histogram of the pseudo random series:

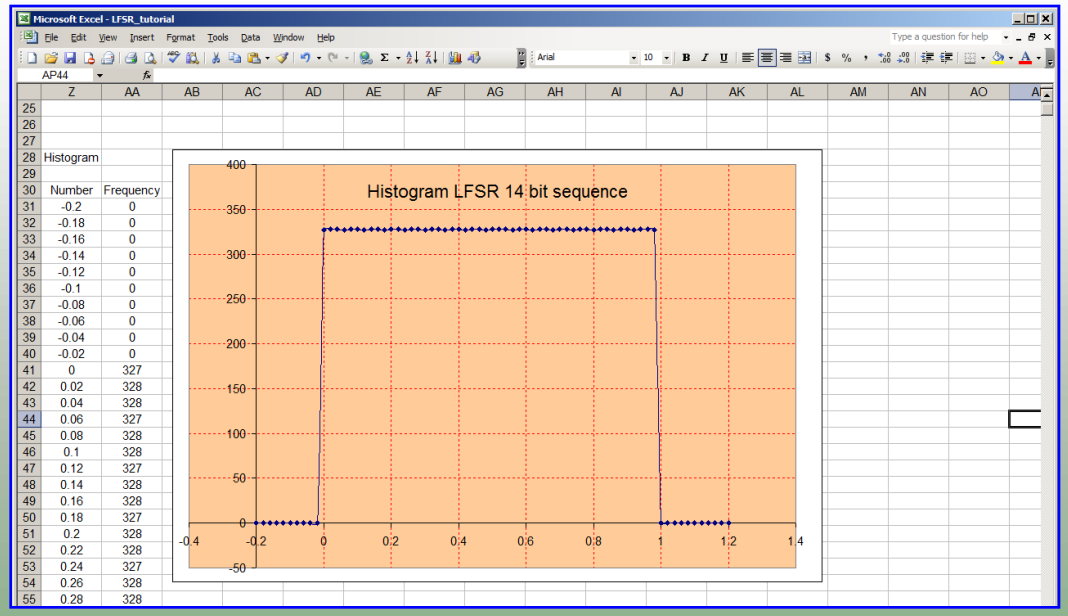
- In statistics, a histogram is a graphical representation showing a visual impression of the distribution of data. It is an estimate of the probability distribution of a continuous variable and was first introduced by Karl Pearson (from Wikipedia).

- The picture to the right shows an example of a histogram in the ordinary form and the cumulative form
- You can see that the x axis of the ordinary histogram represents the outcome of a certain random variable and on the y axis there is the frequency (number of occurrences) of events of the same variable



Implementing a 71-bar histogram of our pseudo random series in the worksheet:

- We will create a 71-bar histogram using worksheet formulas
- Z31: "=-0.2", Z32: "=Z31+0.02" then drag-copy Z32 down to cell Z101, AA31: "=0"
- AA32: "=COUNTIF(R$31:R$16413,"<"&Z33)-COUNTIF(R$31:R$16413,"<"&Z32)"
- Drag-copy AA32 down to cell AA100, then AA101: "=0"
- Chart range Z31:AA101 as a 2D scatter chart
- We can observe the uniform distribution of the variable within the range [0,1).
- We can also verify the uniformity of the distribution just by visual inspection of the histogram table



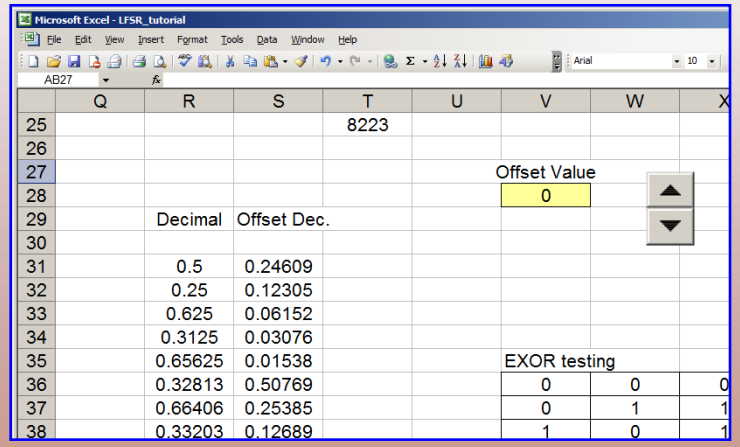
Z	AA
Histogram	
Number	Frequency
-0.2	0
-0.18	0
-0.16	0
-0.14	0
-0.12	0
-0.1	0
-0.08	0
-0.06	0
-0.04	0
-0.02	0
0	327
0.02	328
0.04	328
0.06	327
0.08	328
0.1	328
0.12	327
0.14	328
0.16	328
0.18	327
0.2	328
0.22	328
0.24	327
0.26	328
0.28	328

An interesting idea – the correlation chart:

- The 14-bit LFSR device modeled in this presentation is based on the maximum length linear polynomial and it is supposed to generate $2^{14}-1$ = 16383 different output binary combinations.
- If we split the series of outcomes in half and plot the first half of the series on the x axis and the second half of the series of the y axis of a scatter chart we expect to see a relatively uniform distribution of points across the charting area
- This uniform distribution is expected because the 16383 numbers are different from each other and random
- If the numbers are not perfectly random (the first half is somewhat correlated to the second half) we should see some interesting patterns such as for instance if the series were equal we would see a 45 degree line crossing the chart.
- It would be interesting to insert a small programmable offset between the two groups of data and observe the changes in the chart and any relatively ordered patterns that might arise at certain offset values.

Implementing the correlation chart:

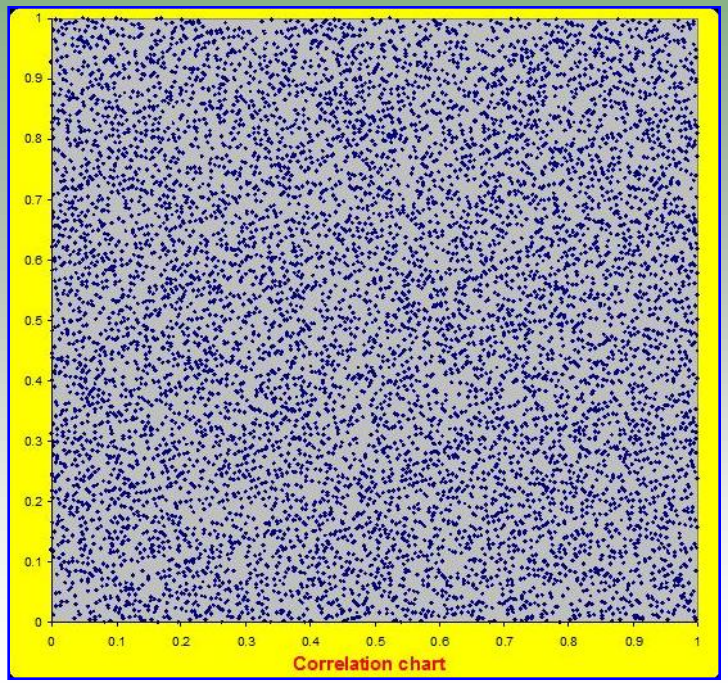
- Cell S31: "=OFFSET(R31,V28+2^13,0)"
- Drag-copy S31 down to cell S8223
- Column S will now contain the second half of the series in column R provided the value in cell V28 is zero
- We can later manipulate the value of cell V28 so that it provides adjustable offset of the data in column S from the second half of the original series (column R)



Microsoft Excel - LFSR_tutorial

	Q	R	S	T	U	V	W	X
25				8223				
26								
27						Offset Value		
28						0		
29		Decimal	Offset Dec.					
30								
31		0.5	0.24609					
32		0.25	0.12305					
33		0.625	0.06152					
34		0.3125	0.03076					
35		0.65625	0.01538			EXOR testing		
36		0.32813	0.50769			0	0	0
37		0.66406	0.25385			0	1	1
38		0.33203	0.12689			1	0	1

- The scatter chart used to display the correlation plot will have the range set from 0 to 1 for both x and y axes and the charted data will be contained in the range P31:S8223



The manual offset macro:

- The following macro, attached to a spin button will control the manual adjustment of the offset in cell V28
- The spin button is set to run between -200 and 200

```
Private Sub Off_Set_Change()
[V28] = Off_Set.Value
End Sub
```

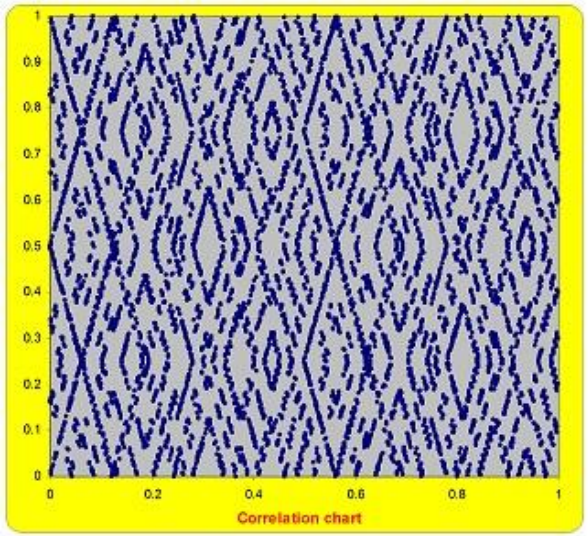
An animated offset macro:

- The macro to the right, attached to a button will control the animated offset routine
- A characteristic of this macro is that it can be started and stopped using the same button. The Boolean variable "abc" serves this purpose.
- The macro uses a conditional "While" loop to increment an integer variable which will be assigned to the offset cell. This way the user can follow on the chart the effect of plotting a half of the random series function on the other half.
- Depending on the offset value these halves might slightly overlap

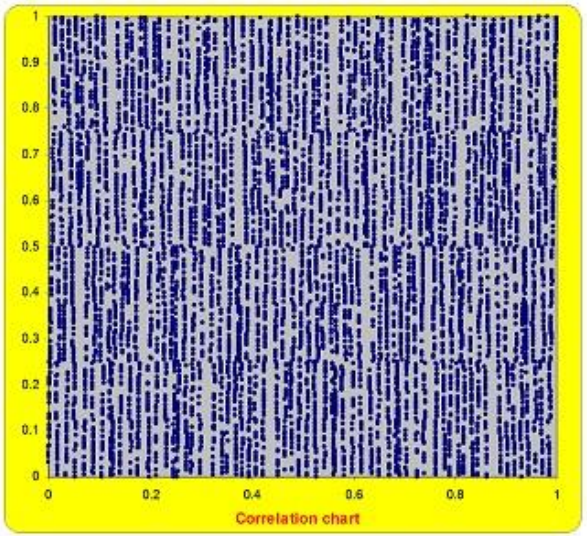
```
Dim abc As Boolean
Dim n As Integer

Sub Animated_Offset_Change()
abc = Not (abc)
   Do While abc = True
   DoEvents
      If n > 200 Then
      n = -200
      Else
      DoEvents
      n = n + 1
      [V28] = n
      End If
   Loop
End Sub
```

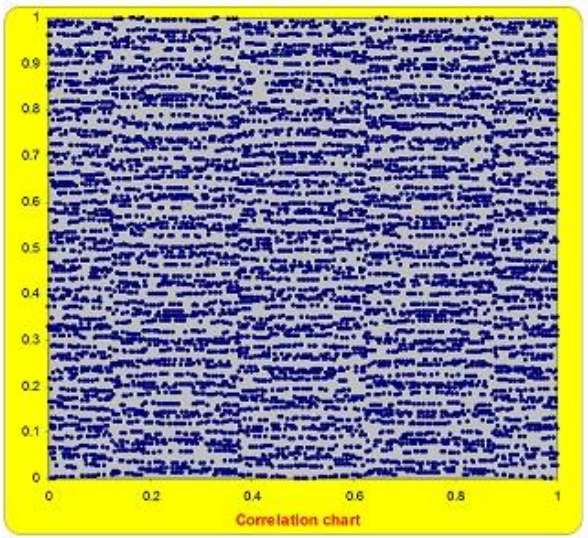
Example of interesting looking correlation charts:



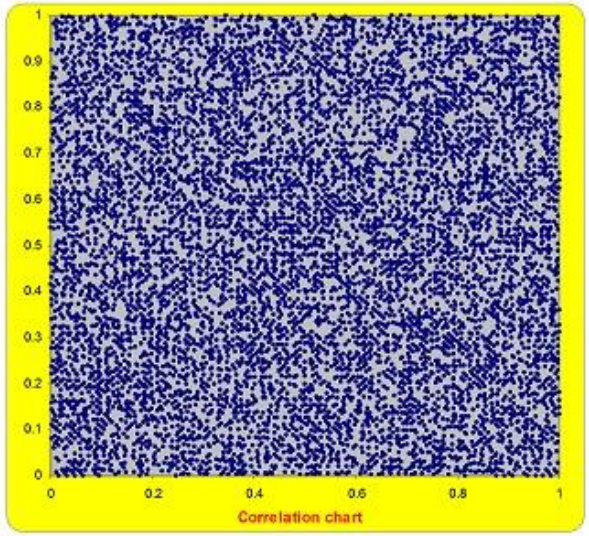
Offset = -149



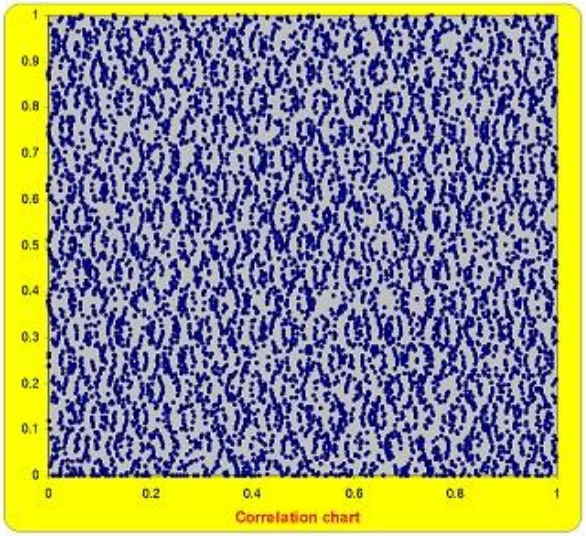
Offset = -116



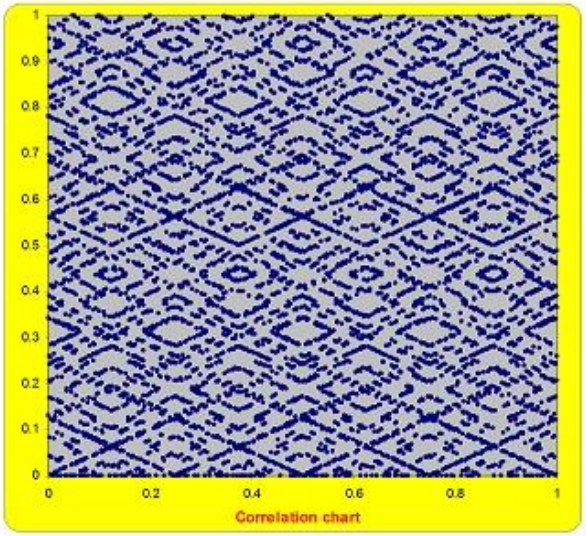
Offset = -78



Offset = 0



Offset = 79



Offset = 147